Proposal of demonstration

of the Goldbach's conjecture

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The Goldbach's conjecture is the unproven mathematic assertion which states : All whole even number greater than 2 can be written as the sum of two prime numbers. It was formulated in 1742 by Christian Goldbach, it is one of the oldest unsolved problem of the theorie of number and mathematics.

1 Preamble

We will agree to say that at the time of writing, Goldbach's conjecture is not proven, that it is verified for all even integers less than $8,875.10^{30}$. And that we don't know why some primes, decompose the even integers in sums of two primes.

2 Installing the tools

In this article to prove the conjecture, I propose to extend the decomposition of integers into prime, by introducing all eligible primes to the decomposition of the even integer considered, which $p_n < n$.

$$n \rightarrow \begin{pmatrix} (n)mod(2) = \cdots \\ (n)mod(3) = \cdots \\ (n)mod(5) = \cdots \\ (n)mod(7) = \cdots \\ \cdots \\ (n)mod(p_n) = \cdots \end{pmatrix} = Signature_n[]$$

From now on, and in the rest of the proposal, I will only take into account the result vector.

$$Sgn_n = []$$

2.1 Minimalist analysis

I can say that the signature is unique, owing to the signature extends or encompasses the prime factor decomposition of the integer.

I can say that if there is no zero in the signature, then n is a prime, owing the signature has all the primes eligible to the decomposition of n, furthermore we know that a composite number always has a factor smaller than its square root.

3 Proposed proof of the Goldbach conjecture

In this first part I present the underlying notion at the origin of the demonstration. To break down an even integer in sums of two prime numbers, it is enough to decompose in sums the values of the signature of $2n$ without introducing zero, then to calculate the integer associated has each signature thus created. As all the integers are decomposable in sums of 2 elements different zero. There is always a solution for $2n = p_a + p_b$.

3.1 Exemple

$$Sgn_{2n} = \begin{pmatrix} (2n)mod(2) = \quad 0 \; = \; 1+1 \\ (2n)mod(3) = \cdots = a_3 + b_3 \\ (2n)mod(5) = \cdots = a_5 + b_5 \\ (2n)mod(7) = \cdots = a_7 + b_7 \\ \cdots \end{pmatrix}$$

So to this decomposition or to its two signatures, I could associate two integers.

$$Sgn_a[] = p_a \quad , \quad Sgn_b[] = p_b$$

If there is no zero present in $Sgn_a[] \quad Sgn_b[]$, then these integers will be prime, this decomposition method is always possible regardless of the value of $2n$, and this method allows to justify the all solutions which decompose $2n$ in sums. It is just necessary not to introduce of zero in $Sgn_a[] \quad Sgn_b[]$ at the levels the factors less than the square root when one decomposes $Sgn_{2n}[]$.

3.2 Proposed proof

So if I consider the signature of a non-decomposable even integer as a sums of two prime numbers. This implies that all primes less than $2n$ modify the signature of $2n$ by imposing a zero at the lower factor levels of the square root of $2n$,$(2n - P_{éligible})mod(p_x) = 0 \quad p_x < \sqrt(2n)$, which means that all eligible primes have a common element in their signature with $2n$, and it is trivially impossible owing all prime numbers have a different signature, cf. Dirichlet's theorem on arithmetic progressions and $\pi(n) \gg \pi(\sqrt{2n})$

3.3 Decomposition method

To calculate the value of the integer from its signature one can use the arithmetic associated with the representation of integers in primorial base.

$$n = \underbrace{\underbrace{\underbrace{\underbrace{\underbrace{1 \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix}}_{(n)mod(2)} + 2 \cdot \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}}_{(n)mod(3)} + 2 \cdot 3 \cdot \begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}}_{(n)mod(5)} + 2 \cdot 3 \cdot 5 \cdot \begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{bmatrix}}_{(n)mod(7)} + 2 \cdot 3 \cdot 5 \cdot 7 \cdot \begin{bmatrix} 0 \\ 1 \\ 2 \\ \cdots \\ 8 \\ 9 \\ 10 \end{bmatrix}}_{\cdots(11)} + \cdots$$

To decompose $2n$, I can also look for a solution by construction so for $2n = 98 \quad \sqrt{98} = 9.89\cdots$. I must therefore construct 2 integers without zero at the levels of the prime factors $2, 3, 5, 7$ of the signature of $Sgn_{98}[]$. Because we know that a compound number always has a factor smaller than its square root.

$$p_a = 1 \cdot [1] \qquad\qquad p_a = 1 \cdot [1] + 2 \cdot [0] = 1$$

$$\begin{array}{cc} (98)mod(2) = \mathbf{0{=}1{+}1} & (98)mod(2) = \mathbf{0{=}1{+}1} \\ (98)mod(3) = 2 = 1 + 1 & (98)mod(3) = \mathbf{2{=}1{+}1} \\ (98)mod(5) = 3 = 1 + 2 & (98)mod(5) = 3 = 1 + 2 \\ (98)mod(7) = 0 = 1 + 6 & (98)mod(7) = 0 = 1 + 6 \\ (98)mod(11) = 10 = 1 + 9 & (98)mod(11) = 10 = 1 + 9 \\ \cdots & \cdots \end{array}$$

$$p_a = 1 \cdot [1] + 2 \cdot [0] + 2 \cdot 3 \cdot [1] = 7 \quad p_a = 1 \cdot [1] + 2 \cdot [0] + 2 \cdot 3 \cdot [1] + 2 \cdot 3 \cdot 5 \cdot [1] = 37$$

$$\begin{array}{cc} (98)mod(2) = \mathbf{0 = 1{+}1} & (98)mod(2) = \mathbf{0 = 1{+}1} \\ (98)mod(3) = \mathbf{2 = 1{+}1} & (98)mod(3) = \mathbf{2 = 1{+}1} \\ (98)mod(5) = \mathbf{3 = 2{+}1} & (98)mod(5) = \mathbf{3 = 2{+}1} \\ (98)mod(7) = 0 = 0 + 0 & (98)mod(7) = \mathbf{0 = 2{+}5} \\ (98)mod(11) = 10 = 7 + 3 & (98)mod(11) = 10 = 4 + 6 \\ \cdots & \cdots \end{array}$$

The addition of Primorial preserves the previous values and allows to build step by step the searched values.It is useless to go beyond the square root, here : $98 = 37 + 61$

3.4 Arithmetic remark

I only found 83 integers, so the largest has a for value of 63274 such that $2n = p_a + p_b$ with $p_b > \sqrt{2n}$, so a prime number less than $\sqrt{2n}$ is sufficient to decompose $2n$, which corroborates Chen Jingrun's theorem.

$$63274 = 293 + 62981 \quad \sqrt{63274} = 251.54...$$

4 Corollary

During the international congress of mathematicians of 1912 in Cambridge, Edmund Landau presented 4 problems which are linked, in this party I propose to apprehend the conjecture of the twin prime numbers and the legend conjecture .

4.1 There is an infinite number of twin primes.

The quantity of elements present in the signature is decorrelated, of the difference between 2 twin primes, triplets, ... owing to.

$$(p_a) mod(2 \cdot 3 \cdot 5 \cdot 7 \cdots) = p_b$$

with the Primorial $\prod p_n \ll p_a$.

4.2 Legendre's conjecture

There exists a prime between n^2 and $(n+1)^2$ for all integers $n \geq 1$,
since $2n = p_a + p_b$ with $p_b <\approx \sqrt{2n}$ then :

$$(n+1)^2 \pm \{0,1\} - p_b = p_a \qquad p_a < (n+1)^2, p_a > n^2$$

5 $\pi(n)$

From the signature of n, I propose to calculate the quantity of combination without zero, to know the value of $\pi(n)$, all the difficulty lies in the upper bound of the product, which remains to be established.

$$Sgn_n = \begin{pmatrix} (n)mod(2) & \in & \{0,1\} \\ (n)mod(3) & \in & \{0,1,2\} \\ (n)mod(5) & \in & \{0,1,2,3,4\} \\ (n)mod(7) & \in & \{0,1,2,3,4,5,6\} \\ \cdots & & \\ (n)mod(p_n) & \in & \{0,1,2,\cdots,p_{(n-1)}\} \\ \cdots & & \end{pmatrix} \pi(n) = \prod_{p=2}^{p=\cdots} (p-1)$$

6 Conclusion

If you don't admit that the conjecture is proven, you can already say that you know why some primes decompose or not the even integers.
Thanks for your attention.

Références

[1] Conjecture de Goldbach.

[2] Conjecture de Legendre.

[3] Conjecture des nombres premiers jumeaux, triplés, quadruplés .

[4] distribution asymptotique des nombres premiers

[5] Primorial numeral system

[6] Largest Known Prime Number: $2^{282,589,933} - 1$

[7] factorisation prime numbers

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[10] Journal of Integer Sequences, Vol. 15

[11] Kvant Selecta: Algebra and Analysis, II: Algebra and Analysis

[12] Un concept très personnel (le dénominateur commun de forme)

[13] Additive Theory of Prime Numbers. Translations of Mathematical Monographs. Vol. 13. Providence, RI : American Mathematical Society

[14] Unique Decomposition Theorem for 3-Manifolds

[15] Dirichlet's theorem on arithmetic progressions

[16] Chen Jingrun