2) $\overline{R}$: "On tire deux boules rouges sans remises"

$$P(\overline{R}) = P(\overline{R} \cap U_1) + P(\overline{R} \cap U_2) + \dots + P(\overline{R} \cap U_n)$$

$$= P(\overline{R} | U_1) \times P(U_1) + \dots + P(\overline{R} | U_n) \times P(U_n)$$

$P(\overline{R} | U_1) = 0$ Car il n'y a qu'une seule boule rouge

$P(\overline{R}|U_2) = \frac{2}{n} \times \frac{1}{n}$; $P(\overline{R}|U_3) = \frac{3}{n} \times \frac{2}{n}$;

$P(\overline{R}|U_{n-1}) = \frac{n-1}{n} \times \frac{n-2}{n}$; $P(\overline{R}|U_n) = 1$

$$\Rightarrow \forall k \in [\![2; n-1]\!] \quad P(\overline{R}|U_k) = \frac{k(k-1)}{n^2}$$

$$\Rightarrow P(\overline{R}) = \frac{1}{n}\left(\sum_{k=1}^{n-1} \frac{k(k-1)}{n^2} + 1\right)$$

$$\Rightarrow P(\overline{R}) = \frac{1}{n^3}\left(\sum_{k=1}^{n-1} k^2 - \sum_{k=1}^{n-1} k\right) + \frac{1}{n}$$

$$= \frac{1}{n^3}\left[\frac{(n-1)(n)(2n-1)}{6} - \frac{(n-1)(n)}{2}\right] + 1/n$$

$$P(\overline{R}) = \frac{n(n-1)}{n^3}\left[\frac{2n-1-3}{6}\right] + 1/n$$

$$= \frac{(n-1)(n-2)}{3n^2} + \frac{1}{n} = \frac{(n-1)(n-2)+3n}{3n^2}$$

$$P(\overline{R}) = \frac{(n-1)(n-2)+3n}{3n^2}$$

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3) $$P(R) = \frac{(n+1)(2n+1)}{6n^2} \underset{n \to +\infty}{\sim} \frac{2n^2}{6n^2} = \frac{1}{3}$$

$$P(\overline{R}) = \frac{(n-1)(n-2)+3n}{3n^2} \underset{n \to +\infty}{\sim} \frac{n^2}{3n^2} = \frac{1}{3}$$